Comment on "Model for Gravity at Large Distances"

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We correct a sign mistake in the work mentioned in the title; explore consequences on energy conditions in the relevant context, and make a suggestion on the introduced parameter.

Recently Grumiller [1], starting from a simple set of assumptions, proposed the metric[1]

$$ds^2 = -K^2 dt^2 + \frac{dr^2}{K^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad \text{(1a)}$$

$$K^2 = 1 - 2\frac{MG}{r} + 2br, \quad \text{(1b)}$$

as a somewhat general framework to approach various systems with anomalous accelerations such as the rotation curves of spiral galaxies and the Pioneer anomaly. It is stated that b comes in as an arbitrary constant depending on the system under study and that for $b > 0$ and of the order of inverse Hubble length, a qualitative understanding of the mentioned anomalies are possible. It is also stated that the effective energy-momentum tensor resulting from Eqs.(1) is that of an anisotropic fluid obeying the equation of state $p_r = -\rho$ and $p_\theta = p_\phi = p_r/2$ with

$$\rho = \frac{4b}{\kappa r}\,, \quad (2)$$

where κ is the (positive) gravitational coupling constant, i.e. the constant in the Einstein equation $G_{\mu\nu} = \kappa T_{\mu\nu}$.

While we agree on the equation of state we disagree on the sign of ρ; the metric in Eqs.(1) yields[2]

$$\rho = -\frac{4b}{\kappa r}\,. \quad (3)$$

The effective potential formalism for the geodesic equation of a test particle is given by

$$\left(\frac{dr}{d\lambda}\right)^2 + 2V_{\text{eff}}(r) = E^2 - \epsilon, \quad (4)$$

with

$$V_{\text{eff}}(r) = -\epsilon\frac{MG}{r} + \frac{L^2}{2r^2} - MG\frac{L^2}{r^3} + \epsilon br + b\frac{L^2}{r}, \quad (5)$$

where units are chosen such that $c = 1$, λ is the affine parameter along the geodesic and the constants of motion are $E = K^2 dt/d\lambda$ and $L = r^2 d\phi/d\lambda$. Also, for massive and massless particles we have $\epsilon = 1$ and $\epsilon = 0$ respectively.

It is still true that for $b > 0$ the effect of br is a constant anomalous acceleration towards the center for objects moving non-relativistically , *despite the negative energy density*. This follows, because the effective potential is derived from the metric directly; and can also be seen from the Raychaudhuri equation specialized to a collection of test particles initially at rest in a small volume of space [2]:

$$\frac{d\theta}{d\tau} = -4\pi G(\rho + p_r + p_\theta + p_\phi) = 4\pi G\rho\,, \quad (6)$$

where θ is the quantity called *expansion* and τ is the proper-time along geodesics followed by the test particles[3]. The second equality follows from the peculiar equation of state described in [1] and confirmed here. The negativity of the derivative of expansion along geodesics shows that gravity is attractive for a given fluid; this is the case here because of Eq.(3), for positive b.

The negative energy density naturally leads us to question if the fluid violates any of the so-called *energy conditions*[4], the compatibility with which is generally taken as a measure of physical reasonableness. The weak energy condition (WEC) requires $\rho \geq 0$ and $\rho + p_i \geq 0$; the strong energy condition (SEC), $\rho + \sum p_i \geq 0$ and $\rho + p_i \geq 0$; and the dominant energy condition (DEC), $\rho \geq |p_i|$; it is easily seen that our fluid violates all three[5].

On one hand, one might say that the violation of energy conditions means that the model is not very physically reasonable; but on the other hand, we are talking about an *effective* fluid, not necessarily a real one. Also,

[1] We take $\Lambda = 0$ without losing generality of our arguments.

[2] We use MTW [4] sign conventions, but of course the signs of ρ and p's are the same in all commonly used conventions.

[3] The LHS of Eq.6 can also be written as $\ddot{V}/V$ where V denotes the volume occupied by the test particles. See [3] for a nice introduction to general relativity where Raychaudhuri's equation is placed at the center of exposition.

[4] We use the definitions in [2] for the energy conditions.

[5] One can easily find that the null energy condition (NEC) and the null dominant energy condition (NDEC) are also violated.

the attractive nature of the fluid (as confirmed by application of the Raychaudhuri's equation) in the face of these violations serves as an example of a delicate fact about SEC: while SEC ensures that gravity is attractive it does not encompass all *attractive* gravities.

Finally, we would like to point out a possibility for the relation between b and the system under consideration: The fluid is attractive; in fact, the $1/r$ dependence of the density and pressures show that it clusters around the central mass. Though speculative at this point, it seems reasonable that bigger masses will accumulate more fluid, i.e., b will be a monotonically increasing function of M. On the other hand the very meaning of M next to b is questionable because one has to match the metric in Eq.(1) to the metric of the interior system (star or galaxy), the matching conditions will undoubtedly yield a relation between the integral of the energy density inside the interior system which we may call M_s and the parameters of the metric outside; M and b. We leave the quantitative analysis of these points for future work.

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